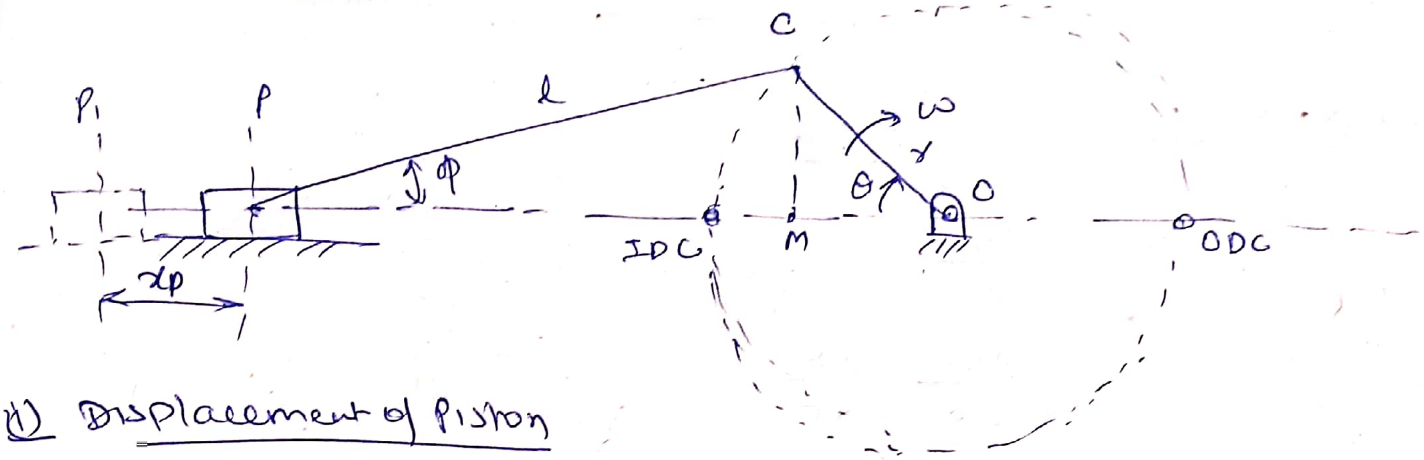
* Analysis of I.C. Engine Mechanism




(i) Displacement of Piston

$x_p = P_1P$

$x_p = P_1O - PO$

$= (l+r) - (PM + MO)$

$= (l+r) - (l\cos\phi + r\cos\theta)$

$= l + r - l\cos\phi - r\cos\theta$

$= l(1-\cos\phi) + r(1-\cos\theta)$

$\because \frac{l}{r} = n$

$\sin\phi = \frac{\sin\theta}{n}$ and $\cos\phi = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$.

$x_p = l\left(1 - \frac{1}{n}\sqrt{n^2-\sin^2\theta}\right) + r(1-\cos\theta)$

$$\boxed{x_p = r\left(n - \sqrt{n^2-\sin^2\theta}\right) + r(1-\cos\theta)}$$

Velocity of Piston

$V_p = \frac{dx_p}{dt} = \frac{dx_p}{d\theta} \times \frac{d\theta}{dt}$

$= \omega \cdot \frac{dx_p}{d\theta}.$

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$$V_p = \omega \left\{ \left[ r \left( \frac{1}{2\sqrt{n^2 - \sin^2\theta}} \cdot (2\sin\theta \cdot \cos\theta) \right) \right] + \left[ r (\sin\theta) \right] \right\}$$

$$\boxed{V_p = r\omega \left[ \sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right]} \quad \text{or} \quad r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

iii) Acceleration of piston

$$a_p = \frac{dV_p}{dt}$$

$$= \frac{dV_p}{d\theta} \times \frac{d\theta}{dt}$$

$$= \omega \cdot \frac{dV_p}{d\theta}$$

$$a_p = r\omega^2 \cdot \frac{d}{d\theta} \left[ \sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right]$$

$$\cancel{a_p = r\omega^2 \left[ \cos\theta + 2\cos 2\theta \right.}$$

let us assume that as

$l > l$ then $n > 1$

here $n$ will be large than $\sin^2\theta$.

So, neglect $\sin^2\theta$.

$$a_p = r\omega^2 \frac{d}{d\theta} \left( \sin\theta + \frac{\sin 2\theta}{2n} \right)$$

1) when crank rotate with uniform angular velocity $\omega$ = constt.

$$a_p = r\omega^2 \left[ \cos\theta + \frac{2\cos 2\theta}{2n} \right]$$

$$\boxed{a_p = r\omega^2 \left[ \cos\theta + \frac{\cos 2\theta}{n} \right]}$$

II) when crank is ~~rod~~ rotating with non-uniform velocity ($\omega \neq$ const)

$$a_p = \omega \cdot \frac{dv_p}{d\theta}$$

$$a_p = \omega \cdot \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right]$$

$$= r\omega \frac{d}{d\theta}\left[\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right]$$

$$= r\omega\left[\frac{d\omega}{d\theta}\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) + \omega\left(\cos\theta + \frac{\cancel{2}\cos 2\theta}{\cancel{2}n}\right)\right]$$

$$= r\omega\left[\frac{d\omega}{dt} \times \frac{dt}{d\theta}\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) + \omega\left(\cos\theta + \frac{\cos 2\theta}{n}\right)\right]$$

$$\boxed{a_p = r\omega\left[\frac{\alpha}{\omega}\left(\sin\theta + \frac{\sin 2\theta}{n}\right) + \omega\left(\cos\theta + \frac{\cos 2\theta}{n}\right)\right]}$$

v) Angular velocity of connecting rod.

$$\sin\phi = \frac{\sin\theta}{n}$$

$$\frac{d}{dt}(\sin\phi) = \frac{d}{dt}\left(\frac{\sin\theta}{n}\right)$$

$$\frac{d\sin\phi}{d\phi} \cdot \frac{d\phi}{dt} = \frac{d}{d\theta}\left(\frac{\sin\theta}{n}\right) \cdot \frac{d\theta}{dt}$$

$$\cos\phi \cdot \omega_c = \frac{\cos\theta}{n} \cdot \omega$$

$$\omega_c = \left(\frac{\cos\theta}{\cos\phi}\right)\frac{\omega}{n}$$

$$\omega_c = \frac{\cos\theta}{\frac{1}{\cancel{n}}\sqrt{n^2 - \sin^2\theta}} \cdot \frac{\omega}{\cancel{n}}$$

$\because n^2 >>> \sin^2\theta$

$\sqrt{n^2} = n.$

$$\boxed{\omega_c = \frac{\omega\cos\theta}{n}}$$

V) Angular Acceleration of connecting rod

$$\omega_c = \frac{\omega \cos\theta}{n}$$

$$\alpha_c = \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\alpha_c = \frac{d}{d\theta}\left(\frac{\omega \cos\theta}{n}\right) \cdot \frac{d\theta}{dt}$$

I when $\omega = \text{Constt}$

$$\alpha_c = \omega \cdot \frac{d}{d\theta}\left(\frac{\omega \cos\theta}{n}\right)$$

~~when~~

$$\alpha_c = \omega^2 \cdot \frac{d}{d\theta}\left(\frac{\cos\theta}{n}\right)$$

$$\boxed{\alpha_c = -\frac{\omega^2 \sin\theta}{n}}$$

II when $\omega \neq \text{Constt}$.

$$\alpha_c = \omega \cdot \frac{d}{d\theta}\left(\frac{\omega \cos\theta}{n}\right)$$

$$\alpha_c = \omega \cdot \left[\frac{d\omega}{d\theta} \cdot \frac{\cos\theta}{n} + \omega \cdot \frac{d}{d\theta}\frac{\cos\theta}{n}\right]$$

$$= \omega\left[\frac{d\omega}{dt} \cdot \frac{dt}{d\theta} \cdot \frac{\cos\theta}{n} - \frac{\omega \sin\theta}{n}\right]$$

$$= \omega\left[\frac{\alpha}{\omega}\frac{\cos\theta}{n} - \frac{\omega \sin\theta}{n}\right]$$

$$\boxed{\alpha_c = \frac{\alpha \cos\theta}{n} - \frac{\omega^2 \sin\theta}{n}}$$

| Sr. No. | Parameter | Equation |
|---|---|---|
| 1. | Displacement of piston | $x_P = r(1-\cos\theta) + r(n-\sqrt{n^2-\sin^2\theta})$ |
| 2. | Velocity of piston | $V_P = \omega \cdot r\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$ |
| 3. | Acceleration of piston<br>a) If angular velocity of crank is uniform<br>b) If angular velocity of crank is not uniform | $a_P = \omega^2 \cdot r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$<br>$a_P = r \cdot \omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) + r \cdot \alpha\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$ |
| 4. | Angular velocity of connecting rod | $\omega_C = \frac{\omega \cdot \cos\theta}{n}$ |
| 5. | Angular acceleration of connecting rod<br>a) If angular velocity of crank is uniform<br>b) If angular velocity of crank is not uniform. | $\alpha_C = \frac{-\omega^2 \cdot \sin\theta}{n}$<br>$\alpha_C = \frac{-\omega^2 \cdot \sin\theta}{n} + \frac{\alpha \cdot \cos\theta}{n}$ |

**Example 7.4 :** *In an I.C. engine mechanism, the stroke of piston is 400 mm and obliquity ratio is 4.5. The crank rotates uniformly at 600 r.p.m. in clockwise direction. Find velocity and acceleration of piston when the crank is approaching I.D.C. and connecting rod is perpendicular to the crank. Also find angular velocity of connecting rod.*

☞ **[ Dec. 2013, 8 Marks ]**

problem 1.4

Given data:-

$2r = 400mm$

$r = 200mm$

$\frac{l}{r} = 4.5$

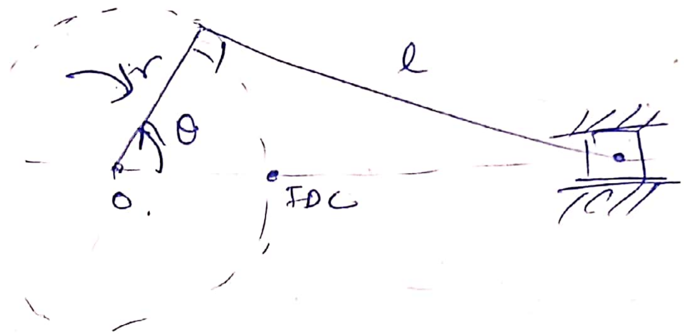
$l = 4.5 \times 200$

$l = 900$

$N = 600 rpm$ (D)

$\omega = \frac{2\pi N}{60}$

$\omega = 62.83$ (D) rad/sec.




$\cancel{\theta = 90^\circ}$

To find?

$v_p = ?$

$a_p = ?$

$\omega_c = ?$

$\tan\theta = \frac{l}{r}$

$\tan\theta = 4.5$

$\theta = 77.47^\circ$

Assume $\omega$ = const

Ans:-

$V_p = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \Rightarrow (0.2)(62.83)\left[\sin \overset{77.47}{\cancel{90}} + \frac{\sin \overset{154.94}{\cancel{180}}}{2(4.5)}\right]$

$\Rightarrow (0.2)(62.83)[1]$

$\boxed{V_p = 12.\cancel{56}\ m/sec}$

$a_p = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$= (0.2)(62.83)^2\left[\cos \overset{77.47}{\cancel{90}} + \frac{\cos \overset{154.94}{\cancel{180}}}{4.5}\right]$

$\boxed{a_p = 12.338\ m/sec^2}$

$\omega_c = \frac{\omega \cos\theta}{n}$

$= \frac{62.83 \cos 77.47}{4.5}$

$\boxed{\omega_c = 3.028\ rad/sec}$

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**Example 7.6 :** *In a slider crank mechanism, lengths of crank and connecting rod are 3 cm and 15 cm respectively. The crank rotates at 600 r.p.m. (clockwise). Find.*

*i) Maximum velocity of slider and corresponding crank position.*

*ii) Acceleration of slider in its extreme positions.*

**[ May 2012, Dec 2012, 8 Marks ]**

7.6 Problem

Given :-

$l = 15cm = 0.15m$

$r = 3cm = 0.03m$

$N = 600rpm$ (2)

$\omega = \frac{2\pi N}{60}$

$$\boxed{\omega = 62.83 \text{ rad/sec}}$$

Assume $\omega$ = const

$n = \frac{l}{r} = 5.$

To find :-

$(V_p)_{max} = ?$ @ $\theta = ?$

$a_p = ?$ @ $\theta = 0$ and $\theta = 180°$.

Ans :-

$$V_p = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$$

$$\frac{dV_p}{dt} = \frac{dV_p}{d\theta} \times \frac{d\theta}{dt}$$

$$= \omega \cdot \frac{dV_p}{d\theta} = 0,$$

$a_p = 0.$

$$r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right] = 0$$

$$\cos\theta + \frac{\cos 2\theta}{n} = 0$$

$$\cancel{\frac{\cos 2\theta}{n} = -\cos\theta}$$

$$\cos\theta + \frac{(2\cos^2\theta - 1)}{n} = 0$$

$$n\cos\theta + 2\cos^2\theta - 1 = 0$$

$$2\cos^2\theta + n\cos\theta - 1 = 0$$

$$2\cos^2\theta + 5\cos\theta - 1 = 0$$

$$\cos\theta = 0.186$$

$\theta = 79.28°$ or ~~280~~.72

$360 - 79.28° = 280.72$

To calculate maximum velocity

$$(V_P)_{max} = r\omega\left(\sin\theta + \frac{\sin 2\theta}{n}\right)$$

$$= (0.03)(62.83)\left[\sin 79.28 + \frac{\sin(2\times 79.28)}{5}\right]$$

$$\boxed{(V_P)_{max} = 1.921 \text{ m/sec}}$$

Acceleration of slider in extreme position.

$$a_p = r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right]$$

at $\theta = 0°$

$$a_p = (0.03)(62.83)^2\left(\cos 0 + \frac{\cos 0}{n}\right]$$

$$\boxed{a_p = 142.118 \text{ m/sec}^2}$$

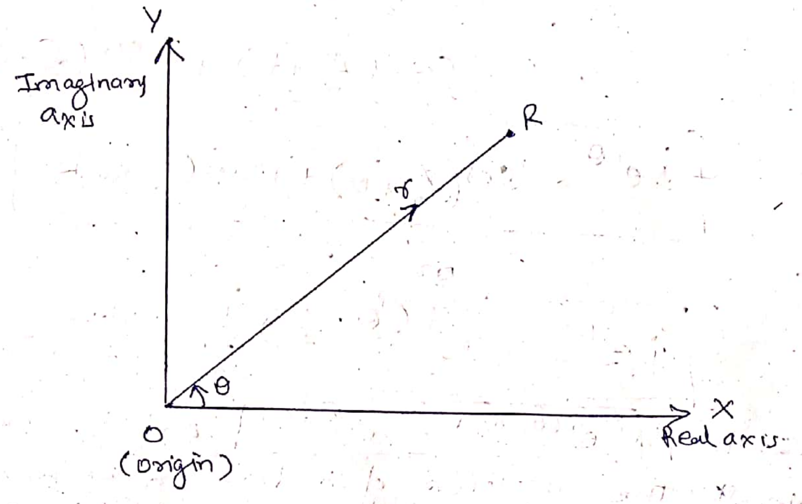
at $\theta = 180°$

$$a_p = (0.03)(62.83)^2\left[\cos 180 + \frac{\cos 360}{5}\right)$$

$$\boxed{a_p = -94.745 \text{ m/sec}^2}$$

## * Complex Algebra method:-

- It is also called Complex variable method.
- Complex number represents kinematic vectors.
- A vector is represented by its magnitude and direction in polar form.
- Figure shows position vector of link OR or position of point R.




- magnitude of link OR = $r$
  direction —"— —"— = $\theta$
- In polar form,
  $$\vec{OR} = \vec{R} = r\angle\theta$$
- Remember, complex numbers are not vector but they can be used to represent the vectors in a plane. Representation of complex number is done by
  x-axis → Real axis
  y-axis → Imaginary axis.
- Polar form of equn in complex algebra is given by,
  $$\boxed{\vec{R} = r \cdot e^{i\theta}}$$

where,
$r$ = magnitude of $\vec{R}$
$\theta$ = Direction of $\vec{R}$ w.r.t real axis
$i = \sqrt{-1}$

Similarly expanded form of $r.e^{i\theta}$ is.
$$\boxed{r.e^{i\theta} = r[\cos\theta + i\sin\theta]}$$

Relations in complex algebra:-

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

Similarly,

$$i \cdot e^{i\theta} = i\cos\theta + i^2 \sin\theta$$

$$= i\cos\theta - \sin\theta \qquad \because i^2 = -1$$

$$= i\sin\left(\frac{\pi}{2}+\theta\right) + \cos\left(\frac{\pi}{2}+\theta\right)$$

$$= \cos\left(\frac{\pi}{2}+\theta\right) + i\sin\left(\frac{\pi}{2}+\theta\right)$$

$$\boxed{i \cdot e^{i\theta} = \cos\left(\frac{\pi}{2}+\theta\right) + i\sin\left(\frac{\pi}{2}+\theta\right)}$$

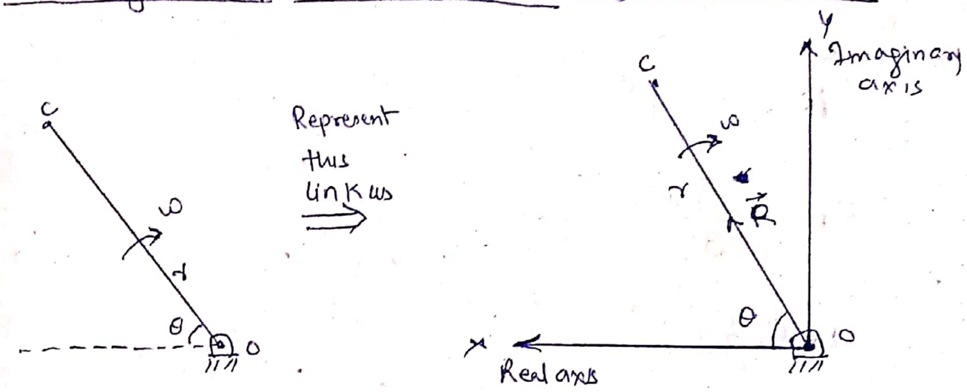
or

$$\boxed{i \cdot e^{i\theta} = e^{i\left(\theta+\frac{\pi}{2}\right)}}$$

Similarly $\boxed{i \cdot e^{i(\theta+\pi/2)} = e^{i(\theta+\pi)}}$ and so on - - - -

note→ The position of a link in 4-bar mechanism, slider crank mechanism is shown by position vector (i.e $r \cdot e^{i\theta}$) and on differentiating it, we will get velocity vector and further differentiation will give acceleration.

# Velocity Analysis by Complex Algebra Method:-




Position vector of OC link,

$$\boxed{\vec{R} = r.e^{i\theta}}$$

velocity vector of OC link.

$$\vec{V}_R = \frac{d\vec{R}}{dt} = \frac{d}{dt}(r.e^{i\theta})$$

$$= r.\frac{d}{dt}e^{i\theta},$$

$$\vec{V}_R = r\left[e^{i\theta}.\frac{d}{dt}.(i\theta)\right]$$

$$= r.e^{i\theta}.i\frac{d\theta}{dt}$$

$$\vec{V}_R = r.\omega.i.e^{i\theta}$$

$$\vec{V}_R = r\omega\,(i.e^{i\theta})$$

$$\boxed{\vec{V}_R = r\omega\left[e^{i(\theta+\frac{\pi}{2})}\right]}$$

In vector $\vec{V}_R$, $r\omega \rightarrow$ magnitude

$e^{i(\theta+\frac{\pi}{2})} \rightarrow$ direction.

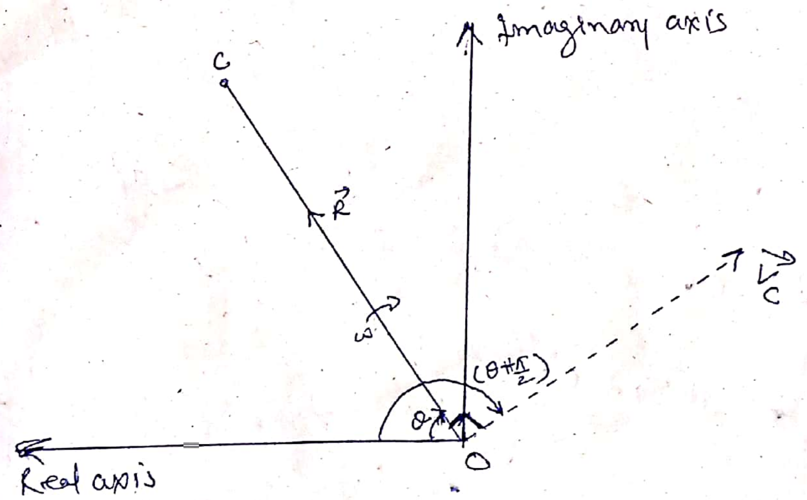



Fig :- velocity vector Representation.

## * Acceleration Analysis by Complex Algebra Method:-

$\therefore \vec{V}_R = r \cdot \omega \left[ e^{i(\theta + \frac{\pi}{2})} \right]$

$\vec{a}_R = \frac{d}{dt}(\vec{V}_R)$

$= \frac{d}{dt} \left\{ r\omega \cdot \left[ e^{i(\theta+\frac{\pi}{2})} \right] \right\}$

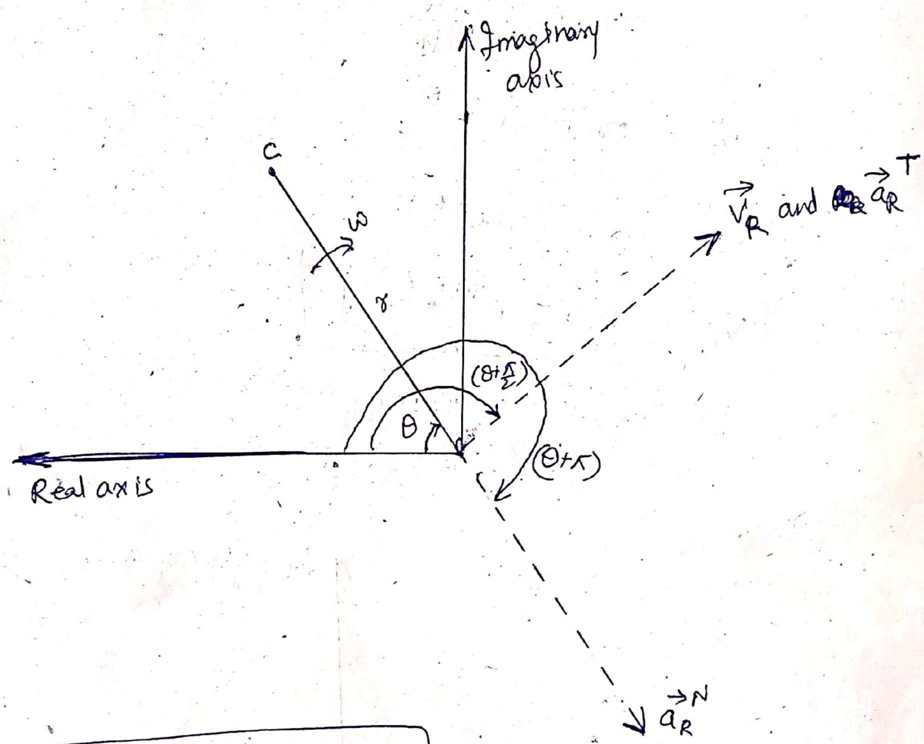
$= r \left\{ \frac{d}{dt} \omega \cdot e^{i(\theta+\frac{\pi}{2})} \right\}$

$\vec{a}_R = r \cdot \left[ \omega \cdot e^{i(\theta+\frac{\pi}{2})} \cdot i\frac{d\theta}{dt} + e^{i(\theta+\frac{\pi}{2})} \cdot \frac{d\omega}{dt} \right]$

$= r \left[ \omega^2 i e^{i(\theta+\frac{\pi}{2})} + \alpha \cdot e^{i(\theta+\frac{\pi}{2})} \right]$

$= r \left[ \omega^2 \cdot e^{i(\theta+\pi)} + \alpha \cdot e^{i(\theta+\frac{\pi}{2})} \right]$

$$\boxed{\vec{a}_R = r\omega^2 \cdot e^{i(\theta+\pi)} + r\alpha \cdot e^{i(\theta+\frac{\pi}{2})}}$$




$$\boxed{\begin{aligned} \vec{a}_R^N &= r \cdot \omega^2 \cdot e^{i(\theta+\pi)} \\ \vec{a}_R^T &= r\alpha \cdot e^{i(\theta+\frac{\pi}{2})} \end{aligned}}$$

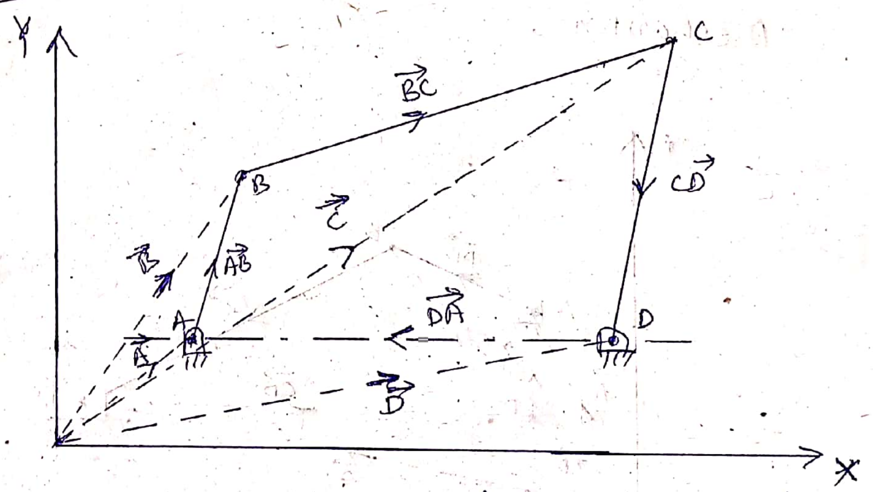
## * Loop-Closure Equation:-

"The sum of the relative position vectors for the links forming a closed loop in the mechanism taken by order is always zero".

This is known as loop-closure equation

- Loop-closure equation gives relation between position vectors forming a closed loop in mechanism.
- The links forming closed loop in the mechanism are rigid, hence the successive joints of the links always remain at fixed distance from each other.
- So, even loop changes its position, magnitude of relative position vector don't change w.r.t time.

## I Loop-Closure Equation for 4-bar Mechanism:-




$\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are position vectors for point A, B, C and D resp.

Rel. position of $\vec{B}$ w.r.t $\vec{A}$ = $\vec{AB} = \vec{B} - \vec{A}$

— " — " — $\vec{C}$ — " — B = $\vec{BC} = \vec{C} - \vec{B}$

— " — " — D — " — C = $\vec{CD} = \vec{D} - \vec{C}$

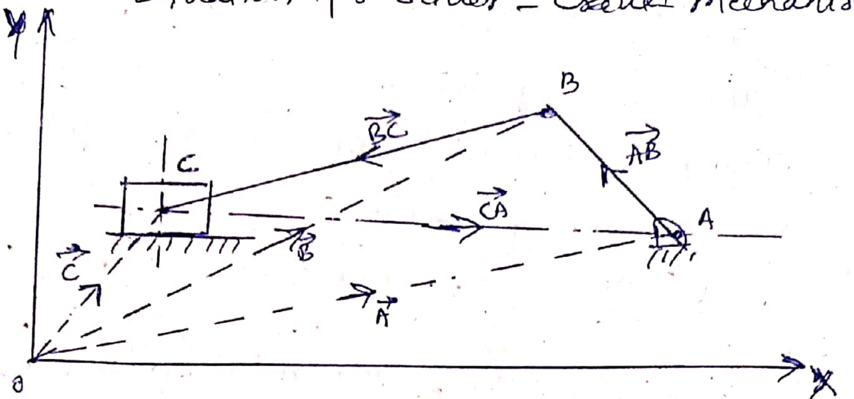
— " — " — A — " — D = $\vec{DA} = \vec{A} - \vec{D}$

By adding Rel. Position vectors,

$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA} = \cancel{\vec{B}} - \cancel{\vec{A}} + \cancel{\vec{C}} - \cancel{\vec{B}} + \cancel{\vec{D}} - \cancel{\vec{C}} + \cancel{\vec{A}} - \cancel{\vec{D}}$$
$$= 0$$

Hence this is loop closure eqtn for 4-bar Mechanism.

II Loop closure Equation for slider - crank mechanism.




Position vector for point A, B, and C are $\vec{A}$, $\vec{B}$ and $\vec{C}$ respectively.

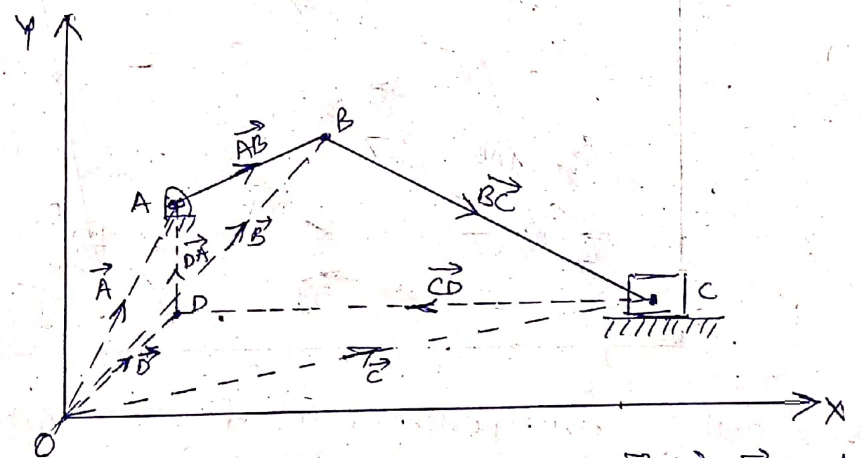
Adding Relative vectors

$$\vec{AB} + \vec{BC} + \vec{CA} = (\vec{B} - \vec{A}) + (\vec{C} - \vec{B}) + (\vec{A} - \vec{C})$$

$$= \cancel{\vec{B}} - \cancel{\vec{A}} + \cancel{\vec{C}} - \cancel{\vec{B}} + \cancel{\vec{A}} - \cancel{\vec{C}}$$

$$\boxed{\vec{AB} + \vec{BC} + \vec{CA} = 0}$$

III Loop closure Equation for offset slider crank mechanism.




Position vectors of points A, B, C, and D are $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$

Adding relative vectors

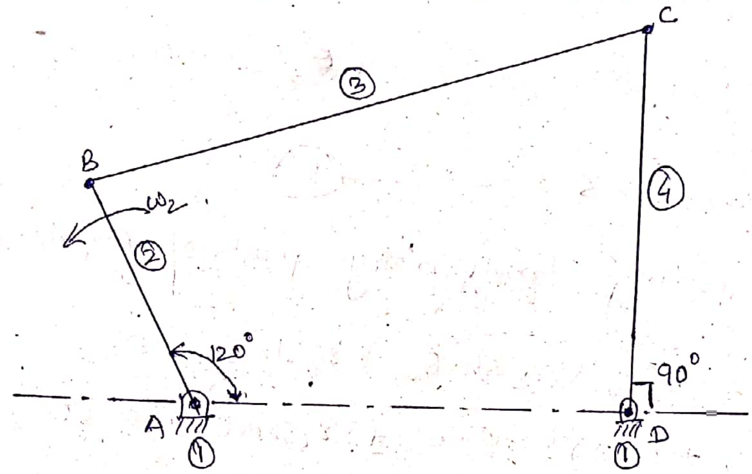
$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA} = (\vec{B} - \vec{A}) + (\vec{C} - \vec{B}) + (\vec{D} - \vec{C}) + (\vec{A} - \vec{D})$$

$$= \cancel{\vec{B}} - \cancel{\vec{A}} + \cancel{\vec{C}} - \cancel{\vec{B}} + \cancel{\vec{D}} - \cancel{\vec{C}} + \cancel{\vec{A}} - \cancel{\vec{D}}$$

$$\boxed{\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA} = 0}$$

Ques. The four bar mechanism ABCD is shown in fig. which is driven by link 2 at $\omega_2 = 45$ rad/sec (↺). Find the angular velocities of link 3 and 4 using Complex number method.

AB = 100 mm, CD = 300 mm, AD = 250 mm.




Ans Given:-

$\omega_2 = 45$ rad/sec (↺).

$\omega_3 = ?$

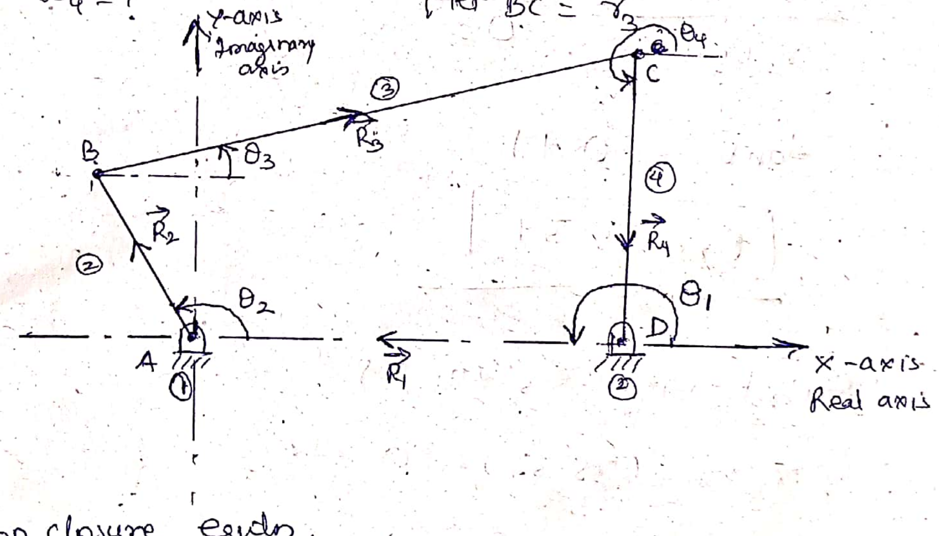
$\omega_4 = ?$

AB = 100 mm = 0.1 m = $r_2$

CD = 300 mm = 0.3 m = $r_4$.

AD = 250 mm = 0.25 m = $r_1$

let BC = $r_3$




loop closure equations,

$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA} = 0.$$

$$\vec{R_1} + \vec{R_2} + \vec{R_3} + \vec{R_4} = 0$$

$$r_1 e^{i\theta_1} + r_2 . e^{i\theta_2} + r_3 e^{i\theta_3} + r_4 e^{i\theta_4} = 0 \quad \text{——— (a)}$$

$$r_1[\cos\theta_1 + i\sin\theta_1] + r_2[\cos\theta_2 + i\sin\theta_2] + r_3[\cos\theta_3 + i\sin\theta_3]$$
$$+ r_4[\cos\theta_4 + i\sin\theta_4] = 0$$

$$r_1\cos\theta_1 + ir_1\sin\theta_1 + r_2\cos\theta_2 + ir_2\sin\theta_2 + r_3\cos\theta_3 + ir_3\sin\theta_3$$
$$+ r_4\cos\theta_4 + ir_4\sin\theta_4 = 0$$

$(r_1 \cos\theta_1 + r_2 \cos\theta_2 + r_3 \cos\theta_3 + r_4 \cos\theta_4) + i(r_1 \sin\theta_1 + r_2 \sin\theta_2$
$+ r_3 \sin\theta_3 + r_4 \sin\theta_4) = 0$ ——— (1)

Equating real parts of equtn (1)

$r_1 \cos\theta_1 + r_2 \cos\theta_2 + r_3 \cos\theta_3 + r_4 \cos\theta_4 = 0$

$(0.25) \cos(180) + (0.1) \cos(120) + r_3 \cos\theta_3 + 0.3 \cos(270) = 0$

$r_3 \cos\theta_3 = 0.3$ ——— (2)

Similarly equating imaginary parts of equtn (1)

$r_1 \sin\theta_1 + r_2 \sin\theta_2 + r_3 \sin\theta_3 + r_4 \sin\theta_4 = 0$

$(0.25) \sin(180) + (0.1) \sin(120) + r_3 \sin\theta_3 + 0.3 \cos 270 = 0$

$r_3 \sin\theta_3 = 0.213$ ——— (3)

Dividing equtn (3) by (2).

$$\frac{\cancel{r_3} \sin\theta_3}{\cancel{r_3} \cos\theta_3} = \frac{0.213}{0.3}$$

$\tan\theta_3 = 0.71$

$\boxed{\theta_3 = 35.37^\circ}$

Substituting $\theta_3$ in equtn (3)

$r_3 \sin(35.37) = 0.213$

$\boxed{r_3 = 0.368 m}$

Now differentiating equtn (a) ~~by~~ w.r.t time.

$\frac{d}{dt}(r_1 e^{i\theta_1} + r_2 \cdot e^{i\theta_2} + r_3 e^{i\theta_3} + r_4 e^{i\theta_4}) = 0$

$r_1 \cdot e^{i\theta_1} \cdot i \frac{d\theta_1}{dt} + r_2 e^{i\theta_2} \cdot i \frac{d\theta_2}{dt} + r_3 \cdot e^{i\theta_3} i \frac{d\theta_3}{dt} + r_4 \cdot e^{i\theta_4} \cdot i \frac{d\theta_4}{dt} = 0$

$r_1 \cdot \omega_1 \cdot i \cdot e^{i\theta_1} + r_2 \omega_2 \cdot i \cdot e^{i\theta_2} + r_3 \omega_3 \cdot i \cdot e^{i\theta_3} + r_4 \cdot \omega_4 \cdot i \cdot e^{i\theta_4} = 0$

$\therefore \omega_1 = 0$. — as link is fix.

$$r_2 \omega_2 \cdot i \cdot e^{i\theta_2} + r_3 \omega_3 \cdot i \cdot e^{i\theta_3} + r_4 \cdot \omega_4 \cdot i \cdot e^{i\theta_4} = 0$$

$$r_2 \omega_2 i \left[\cos\theta_2 + i\sin\theta_2\right] + r_3 \omega_3 i \left[\cos\theta_3 + i\sin\theta_3\right] + r_4 \omega_4 i \left[\cos\theta_4 + i\sin\theta_4\right] = 0$$

$$r_2 \omega_2 i \cos\theta_2 + r_2 \omega_2 i^2 \sin\theta_2 + r_3 \omega_3 i \cos\theta_3 + r_3 \omega_3 i^2 \sin\theta_3$$
$$+ r_4 \omega_4 i \cos\theta_4 + r_4 \omega_4 i^2 \sin\theta_4 = 0$$

$$r_2 \omega_2 i \cos\theta_2 - r_2 \omega_2 \sin\theta_2 + r_3 \omega_3 i \cos\theta_3 - r_3 \omega_3 \sin\theta_3$$
$$+ r_4 \omega_4 i \cos\theta_4 - r_4 \omega_4 \sin\theta_4 = 0$$

$$-(r_2 \omega_2 \sin\theta_2 + r_3 \omega_3 \sin\theta_3 + r_4 \omega_4 \sin\theta_4)$$
$$+ i(r_2 \omega_2 \cos\theta_2 + r_3 \omega_3 \cos\theta_3 + r_4 \omega_4 \cos\theta_4) = 0. \quad (4)$$

Equating the imaginary parts of equtn (4)

$$r_2 \omega_2 \cos\theta_2 + r_3 \omega_3 \cos\theta_3 + r_4 \omega_4 \cos\theta_4 = 0$$

$$(0.1)(45)\cos(120) + (0.368)(\omega_3)\cos(35.37) + (0.3)(\omega_4)\cos 270 = 0$$

$$0.3\,\omega_3 = 2.25 \qquad \because \cos 270 = 0$$

$$\boxed{\omega_3 = 7.5 \text{ rad/sec}}$$

Now equating real parts of equtn (4)

$$-r_2 \omega_2 \sin\theta_2 - r_3 \omega_3 \sin\theta_3 - r_4 \omega_4 \sin\theta_4 = 0$$

$$-(0.1)(45)\sin 120 - (0.368)(7.5)\sin(35.37)$$
$$- (0.3)(\omega_4)\sin 270 = 0$$

$$\boxed{\omega_4 = 18.313 \text{ rad/sec}}$$

Ques In slider Crank mechanism, Crank radius = 100mm, length of Connecting rod = 500mm, Crank angular velocity = 15 rad/sec (↻), The angular acceleration of Crank = 115 rad/sec$^2$. Find.

① Acceleration of Piston

② Angular acceleration of C.R. when Crank is $60^\circ$ from IDC.

Ans

Given:-

$r = 100mm = 0.1m$

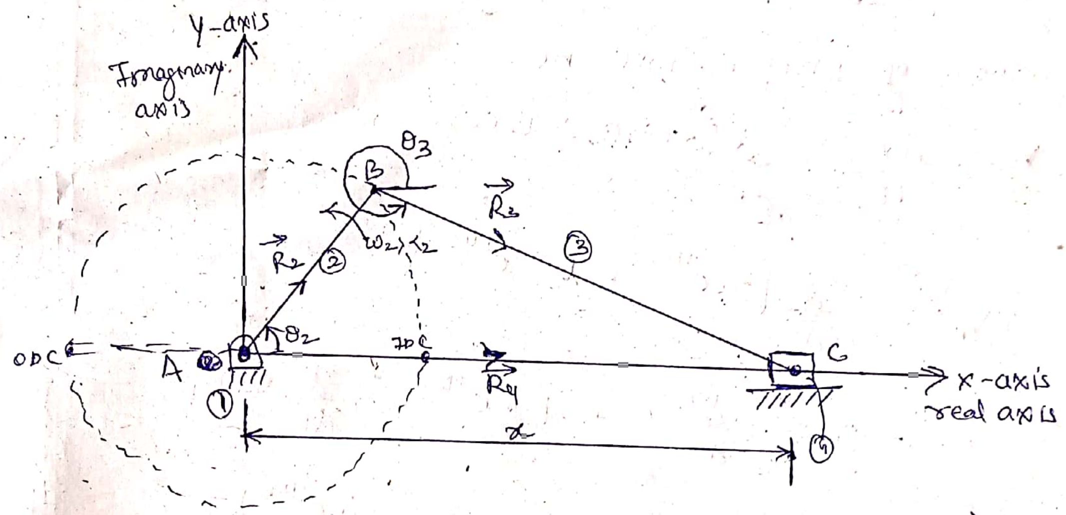
$L = 500mm = 0.5m$

$\omega_2 = 15$ rad/sec (↻)

$\alpha_2 = 115$ rad/sec$^2$

$a_4$ or $a_p$ = ?

$\alpha_3$ or $\alpha_C$ = ? at $60^\circ$ from IDC




For the mechanism, the loop-closure equtn in Complex number form is given by.

$$\vec{AB} + \vec{BC} - \vec{AC} = 0$$

$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\vec{R_2} + \vec{R_3} = \vec{AC}$$

$$r.e^{i\theta_2} + L.e^{i\theta_3} = x.e^{i(0^\circ)}$$

$$r.e^{i\theta_2} + L.e^{i\theta_3} = x$$

$$r[\cos\theta_2 + i\sin\theta_2] + L[\cos\theta_3 + i\sin\theta_3] = x$$

$$(r\cos\theta_2 + L\cos\theta_3) + i[r\sin\theta_2 + L\sin\theta_3] = x + i(0) \quad —①$$

Equating the real and Imaginary parts of equtn

Equating imaginary parts first,

~~r cos θ₂ + ...~~

$$r\sin\theta_2 + L\sin\theta_3 = 0$$

$$(0.1)\sin(60) + (0.5)\sin\theta_3 = 0$$

$$0.5\sin\theta_3 = -0.0866$$

$$\boxed{\theta_3 = -9.962^\circ}$$

or

$$\theta_3 = 360 - 9.962^\circ$$

$$\boxed{\theta_3 = 350.038^\circ}$$

~~Angular velocity of connecting rod~~

equating real parts of equn ①

$$r\cos\theta_2 + L\cos\theta_3 = x \quad \text{— displacement of Piston.}$$

velocity of piston will be,

$$V_p = \frac{dx}{dt} = \frac{d(r\cos\theta_2 + L\cos\theta_3)}{dt}$$

$$V_p = \frac{d}{dt}\left[r.e^{i\theta_2} + L.e^{i\theta_3}\right]$$

$$V_p = \left[r.e^{i\theta_2}.i\frac{d(\theta_2)}{dt} + L.e^{i\theta_3}.i.\frac{d(\theta_3)}{dt}\right]$$

$$V_p = r.\omega_2(i.e^{i\theta_2}) + L.\omega_3(i.e^{i\theta_3})$$

$$\cancel{r\omega_2 e^{i(\theta_2+\frac{\pi}{2})} + L\omega_3 e^{i(\theta_3+\frac{\pi}{2})}}$$

$$V_p = r\omega_2 i(\cos\theta_2 + i\sin\theta_2) + L\omega_3 i(\cos\theta_3 + i\sin\theta_3)$$

$$V_p = r\omega_2.i\cos\theta_2 + i^2 r\omega_2\sin\theta_2 + L\omega_3 i\cos\theta_3 + i^2 L\omega_3\sin\theta_3$$

$$V_p = r\omega_2.i\cos\theta_2 - r\omega_2\sin\theta_2 + L\omega_3 i\cos\theta_3 - L\omega_3\sin\theta_3$$

$$V_p = -(r\omega_2\sin\theta_2 + L\omega_3\sin\theta_3) + i(r\omega_2\cos\theta_2 + L\omega_3\cos\theta_3) \quad ②$$

Equating the imaginary parts of equn ②

$$r\omega_2\cos\theta_2 + L\omega_3\cos\theta_3 = 0$$

$$r\omega_2\cos\theta_2 + L\omega_3\cos\theta_3 = 0$$

$$(0.1)(15)\cos(60) + (0.5)\,\omega_3\cos(350.05) = 0$$

$$\omega_3 = -1.522 \text{ rad/sec } (3)$$

Assuming that the connecting rod is rotating at Anticlockwise direction but the value of $\omega_3$ is -ve so it is actually rotating with clockwise motion.

$$\boxed{\omega_3 = 1.522 \text{ rad/sec } (2)}$$

Now differentiating equn (2) w.r.t time.

$$a_p = \frac{d(V_p)}{dt}$$

$$= \frac{d}{dt}\left[r\omega_2(i\cdot e^{i\theta_2}) + L\omega_3(i\cdot e^{i\theta_3})\right]$$

$$= r\cdot i\left[\frac{d}{dt}(\omega_2\cdot e^{i\theta_2})\right] + L\cdot i\left[\frac{d}{dt}(\omega_3\cdot e^{i\theta_3})\right]$$

$$= r\cdot i\left[\omega_2\cdot\frac{d}{dt}\cdot e^{i\theta_2} + e^{i\theta_2}\cdot\frac{d\omega_2}{dt}\right] + Li\left[\omega_3\cdot\frac{d}{dt}e^{i\theta_3} + e^{i\theta_3}\cdot\frac{d\omega_3}{dt}\right]$$

$$a_p = r\cdot i\left[\omega_2\cdot e^{i\theta_2}\cdot i\frac{d\theta_2}{dt} + e^{i\theta_2}\cdot\alpha_2\right] + Li\left[\omega_3\cdot e^{i\theta_3}\cdot i\frac{d\theta_3}{dt} + e^{i\theta_3}\cdot\alpha_3\right]$$

$$a_p = r i\left[\omega_2^2\cdot i\cdot e^{i\theta_2} + \alpha_2\cdot e^{i\theta_2}\right] + Li\left[\omega_3^2\cdot i\cdot e^{i\theta_3} + \alpha_3\cdot e^{i\theta_3}\right]$$

$$a_p = r i^2\omega_2^2\cdot e^{i\theta_2} + r\cdot i\cdot\alpha_2\cdot e^{i\theta_2} + Li^2\omega_3^2\cdot e^{i\theta_3} + Li\cdot\alpha_3\cdot e^{i\theta_3}$$

$$a_p = -r\omega_2^2\cdot e^{i\theta_2} + i\cdot r\cdot\alpha_2\cdot e^{i\theta_2} - L\cdot\omega_3^2\cdot e^{i\theta_3} + iL\cdot\alpha_3\cdot e^{i\theta_3}$$

$$a_p = -\left[r\omega_2^2\cdot e^{i\theta_2} + L\cdot\omega_3^2\cdot e^{i\theta_3}\right] + i\left[r\alpha_2 e^{i\theta_2} + L\cdot\alpha_3\cdot e^{i\theta_3}\right]$$

$$a_p = -\left[r\omega_2^2(\cos\theta_2 + i\sin\theta_2) + L\omega_3^2(\cos\theta_3 + i\sin\theta_3)\right] + i\left[r\alpha_2(\cos\theta_2 + i\sin\theta_2) + L\alpha_3(\cos\theta_3 + i\sin\theta_3)\right]$$

$$\cancel{a_p = (-r\omega_2^2\cos\theta_2 - ir\omega_2^2\sin\theta_2 + L\omega_3^2\cos\theta_3 + iL\omega_3^2\sin\theta_3) + ir\alpha_2\cos\theta_2}$$

$$a_p = -r\omega_2^2\cos\theta_2 - r\omega_2^2 i\sin\theta_2 - L\omega_3^2\cos\theta_3 - iL\omega_3^2\sin\theta_3 + ir\alpha_2\cos\theta_2 + iL\alpha_3\cos\theta_3 + i^2L\alpha_3\sin\theta_3 + i^2 r\alpha_2\sin\theta_2$$

$$a_p = -r\omega_2^2\cos\theta_2 - L\omega_3^2\cos\theta_3 - r\alpha_2\sin\theta_2 - L\alpha_3\sin\theta_3 - r\omega_2^2 i\sin\theta_2 - L\omega_3^2 i\sin\theta_3 + r\alpha_2 i\cos\theta_2 + iL\alpha_3\cos\theta_3$$

$$a_p = (-r\omega_2^2 \cos\theta_2 - L\omega_3^2 \cos\theta_3 - r\alpha_2 \sin\theta_2 - l\alpha_3 \cos\theta_3)$$
$$+ i(-r\omega_2^2 \sin\theta_2 - l\omega_3^2 \sin\theta_3 + r\alpha_2 \cos\theta_2 + l\alpha_3 \cos\theta_3) \longrightarrow (3)$$

Equating imaginary part of equtn (3).

$$-r\omega_2^2 \sin\theta_2 - l\omega_3^2 \sin\theta_3 + r\alpha_2 \cos\theta_2 + l\alpha_3 \cos\theta_3 = 0$$

$$-0.1(15)^2 \sin(60) - (0.5)(-1.522)^2 \sin(350.038)$$
$$+ (0.1)(115)\cos(60) + (0.5)(\alpha_3)\cos(350.038) = 0$$

$$\boxed{\alpha_3 = 27.484 \text{ rad/sec}^2}$$

Similarly equating real part of equtn (3).

$$a_p = -r\omega_2^2 \cos\theta_2 - L\omega_3^2 \cos\theta_3 - r\alpha_2 \sin\theta_2 - l\alpha_3 \cos\theta_3$$

$$a_p = -(0.1)(15)^2 \cos 60 - (0.5)(-1.522)^2 \cos(350.038)$$
$$-(0.1)(115)\sin(60) - (0.5)(27.48)\cos(350.038)$$

$$\boxed{a_p = -19.972 \text{ m/sec}}$$

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## * vector Algebra method :-


### 1) velocity Analysis:-

$\vec{R}$ = Rotating vector

ω = Angular velocity of $\vec{R}$

Differentiating $\vec{R}$ w.r.t time

$$\frac{d(\vec{R})}{dt} \Rightarrow \frac{d(R\cdot\hat{R})}{dt}$$

$$\cancel{\hat{R}\cdot\frac{dR}{dt}}$$

$$\frac{d\vec{R}}{dt} \Rightarrow \hat{R}\cdot\frac{dR}{dt} + R\cdot\frac{d\hat{R}}{dt}$$

$$\frac{d\vec{R}}{dt} = \dot{R}\cdot\hat{R} + R\cdot\dot{\hat{R}}$$

But $\dot{\hat{R}} = \omega(\hat{K}\times\hat{R})$

so,

$$\boxed{\frac{d\vec{R}}{dt} = \dot{R}\hat{R} + R\omega(\hat{K}\times\hat{R})}$$

First part is a velocity vector in direction of $\hat{R}$,

Second part is a velocity vector in directn ⊥ar to $\hat{R}$

### 2) Acceleration Analysis:-

$$\vec{V} = \dot{\vec{R}} = \frac{d\vec{R}}{dt}$$

$$\vec{V} = \dot{R}\hat{R} + R\omega(\hat{K}\times\hat{R})$$

if magnitude of R of $\vec{R}$ is constant then

$$\dot{R} = 0$$

so,

$$\boxed{\vec{V} = R\omega(\hat{K}\times\hat{R})}$$

If $\vec{R}$ rotates with uniform velocity, $\omega$ = constt

$$\frac{d\omega}{dt} = \alpha = 0.$$

Now differentiating the velocity equation w.r.t time

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \ddot{\vec{R}}$$

$$\vec{a} = \frac{d}{dt}\left[R\omega\,(\hat{K}\times\hat{R})\right]$$

$$\cancel{= R\omega\left[\hat{K}\times(\omega\times\hat{R})\right]}$$

$$\vec{a} = R\omega\,\frac{d}{dt}\left[\hat{K}\times\hat{R}\right] \qquad \therefore \dot{\hat{R}} = \omega(\hat{K}\times\hat{R})$$

$$\vec{a} = R\omega\left[\hat{K}\times\omega(\hat{K}\times\hat{R})\right]$$

$$\vec{a} = R\omega^2\left[\hat{K}\times(\hat{K}\times\hat{R})\right]$$

$$\therefore \hat{K}\times\hat{K} = 1$$

$$\boxed{\vec{a} = -R\omega^2\,\hat{R}}$$

This acceleration is called as normal or centripetal acceleration which have magnitude $R\omega^2$ and direction opposite to $\hat{R}$.

Similarly if $\omega \neq$ constt.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \ddot{\vec{R}}$$

$$= \frac{d}{dt}\left[R\omega\,(\hat{K}\times\hat{R})\right]$$

$$= (\hat{K}\times\hat{R})\,\frac{d}{dt}(R\omega) + (R\omega)\,\frac{d}{dt}(\hat{K}\times\hat{R})$$

$$\boxed{\vec{a} = (\hat{K}\times\hat{R})\,R\alpha - R\omega^2\,\hat{R}.}$$

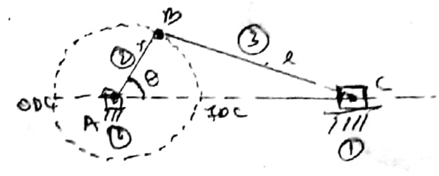
The first part of equation is called as tangential acceleration which have magnitude $R\alpha$ $\perp$ to vector $\hat{R}$.

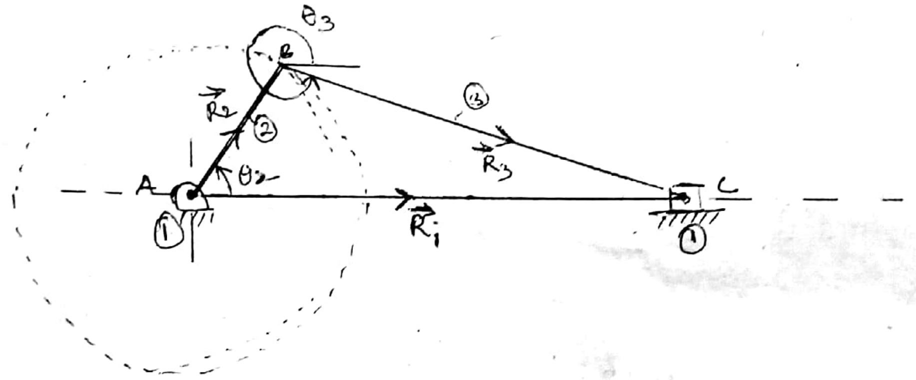
Ques In an I.C Engine Mechanism, Crank is 40 mm long and length of Connecting rod is 160 mm. The Crank angle is 40° with IDC. Find the angle made by Connecting rod with the line of stroke and distance between Crank and piston using vector Algebra.

Ans length of Crank, $r = 40\,mm = 0.04\,m$

length of connecting rod, $l = 160\,mm = 0.16\,m$.

Crank angle, $\theta = 40°$.




By loop closure equtn :-

$$\vec{R_1} - \vec{R_3} - \vec{R_2} = 0 \quad \text{———} \quad ①$$

$$\vec{R_1} = \vec{R_3} + \vec{R_2}$$

$$\therefore \vec{R_1} = R_1 \angle 0°$$

$$\vec{R_2} = r \angle \theta_2 = 0.04 \angle 40°$$

$$\vec{R_3} = l \angle \theta_3 = 0.16 \angle \theta_3$$

Since from equtn ① can be written as

$$R_1 \cdot \hat{R_1} = R_3 \cdot \hat{R_3} + R_2 \cdot \hat{R_2}$$

$$\times \checkmark = \checkmark \times + \checkmark \checkmark$$

$\checkmark \rightarrow$ known
$\times \rightarrow$ not known

So in order to find angular Position of Connecting rod we have to eliminate $R_1$. Hence taking dot product of $(\hat{R_1} \times \hat{K})$ on both sides of equtn ①.

$$\vec{R_1} = \vec{R_3} + \vec{R_2}$$

Taking dot product of $\hat{R_1} \times \hat{K}$ on both sides.

$$\vec{R_1} \cdot (\hat{R_1} \times \hat{K}) = \vec{R_3}\,(\hat{R_1} \times \hat{K}) + \vec{R_2}\,(\hat{R_1} \times \hat{K})$$

$\therefore\ \hat{R_1} = 1 \angle 0^\circ = \hat{i}\cos(0) + \hat{j}\sin(0^\circ)$

$$\boxed{\hat{R_1} = \hat{i}}$$

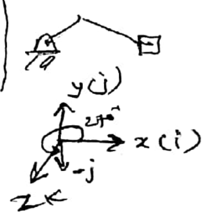
So,

$$R_1 \angle 0^\circ (\hat{i} \times \hat{K}) = 0.16 \angle \theta_3 (\hat{i} \times \hat{K}) + 0.04 \angle 40\ (\hat{i} \times \hat{K})$$

$$R_1 \angle 0^\circ (-\hat{j}) = 0.16 \angle \theta_3 (-\hat{j}) + 0.04 \angle 40\ (-\hat{j})$$

$\therefore\ \hat{i} \times \hat{K} = -\hat{j}$

but $\hat{j} = 1 \angle 270^\circ$




$\therefore\ -\hat{j} = 1 \angle 270^\circ$

as unit vector has magnitude 1 and direction is opposite due to -ve sign hence angle is $270^\circ$

$$(R_1 \angle 0^\circ) \cdot (1 \angle 270) = (0.16 \angle \theta_3) \cdot (1 \angle 270^\circ) + (0.04 \angle 40) \cdot (1 \angle 270)$$

$\therefore\ a \cdot b = a\,b\cos\theta .$

$$R_1 \cos 270 = 0.16\cos(\theta_3 - 270) + 0.04\cos(270 - 40)$$

$$0 = 0.16\cos(\theta_3 - 270) - 0.0257$$

$$0.0257 = 0.16\cos(\theta_3 - 270)$$

$$\cos(\theta_3 - 270) = 0.1606$$

$$\theta_3 - 270 = 80.758^\circ$$

$$\boxed{\theta_3 = 350.75^\circ}$$

Now substituting the value of $\theta_3$ in equation ①

$$\vec{R_1} = \vec{R_2} + \vec{R_3}$$

$$\vec{R_1} \angle 0^\circ = R_2 \angle 40 + R_3 \angle 350.75^\circ$$

$$R_1[\hat{i}\cos 0 + \hat{j}\sin 0] = R_2[\hat{i}\cos 40 + \hat{j}\sin 40] + R_3[\hat{i}\cos 350.75 + \hat{j}\sin 350.75]$$

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$$R_1 \hat{i} = 0.04\left[\hat{i}\cos 40 + \hat{j}\sin 40\right] + 0.16\left[\hat{i}\cos 350.75 + \hat{j}\sin 350.75\right]$$

$$R_1 \hat{i} = 0.0306\,\hat{i} + \cancel{0.0257\,\hat{j}} + 0.1579\,\hat{i} - \cancel{0.0257\,\hat{j}}$$

$$R_1 \hat{i} = 0.1885\,\hat{i}$$

$$\boxed{R_1 = 0.1885\ m}$$

## Computer Aided Analysis for Four Bar Mechanism -

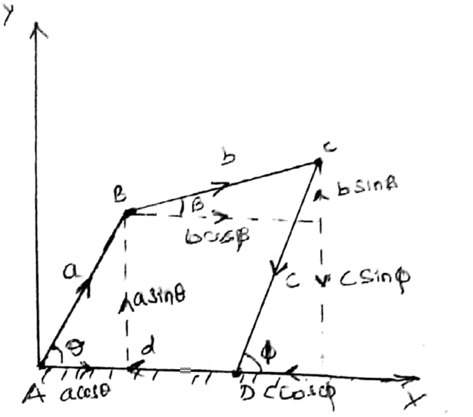


Fig:- Components along x-axis and Y-axis

1. Displacement Analysis:-

(i) For equilibrium of mechanism, sum of the components along X-axis and along Y-axis must be equal to zero.

along x-axis,

$$a\cos\theta + b\cos\beta - c\cos\varphi - d = 0 \quad \text{—— (1)}$$

$$b\cos\beta = c\cos\varphi + d - a\cos\theta .$$

~~along Y-axis~~

squaring both sides

$$b^2\cos^2\beta = (c\cos\varphi + d - a\cos\theta)^2$$

$$b^2\cos^2\beta = c^2\cos^2\varphi + d^2 + a^2\cos^2\theta + 2c.d.\cos\varphi - 2ac.\cos\varphi.\cos\theta - 2a.d\cos\theta \quad \text{—— (2)}$$

Now taking sum of component along Y-axis.

$$a\sin\theta + b\sin\beta - c\sin\varphi = 0 \quad \text{—— (3)}$$

$$b\sin\beta = c\sin\varphi - a\sin\theta$$

squaring both sides.

$$b^2\sin^2\beta = [c\sin\varphi - a\sin\theta]^2$$

$$b^2\sin^2\beta = c^2\sin^2\varphi + a^2\sin^2\theta - 2ac\sin\varphi.\sin\theta \quad \text{—— (4)}$$

Adding eqn (2) and (4)

$$b^2.[\cos^2\beta + \sin^2\beta] = c^2[\cos^2\varphi + \sin^2\varphi] + a^2[\cos^2\theta + \sin^2\theta] + d^2 - 2ac(\cos\varphi\cdot\cos\theta + \sin\varphi\cdot\sin\theta) + 2cd\cos\varphi - 2.a.d\cos\theta$$

$$b^2 = c^2 + a^2 + d^2 + 2cd\cos\varphi - 2ac[\cos\varphi.\cos\theta + \sin\varphi\sin\theta] - 2ad\cos\theta$$

$$2ac[\cos\varphi.\cos\theta + \sin\varphi.\sin\theta] = a^2 - b^2 + c^2 + d^2 + 2cd\cos\varphi - 2a.d\cos\theta$$

$$\cos\varphi.\cos\theta + \sin\varphi.\sin\theta = \frac{(a^2 - b^2 + c^2 + d^2)}{2ac} + \frac{d}{a}\cos\varphi - \frac{d}{c}\cos\theta$$

let $\frac{d}{a} = K_1$, $\frac{d}{c} = K_2$ and $\frac{a^2 - b^2 + c^2 + d^2}{2ac} = K_3$

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$$(\cos\phi \cos\theta + \sin\phi \sin\theta) = K_3 + K_1 \cos\phi - K_2 \cos\theta \quad \text{--- (5)}$$

$$\boxed{\cos(\phi - \theta) = K_1 \cos\phi - K_2 \cos\theta + K_3} \quad \text{--- (6)}$$

This equation is known as Freudenstein's equtn

Since,

$$\sin\phi = \frac{2\tan(\phi/2)}{1+\tan^2(\phi/2)} , \quad \cos\phi = \frac{1-\tan^2(\phi/2)}{1+\tan^2(\phi/2)}$$

Substituting the values in equtn (5).

$$\left[\frac{1-\tan^2(\phi/2)}{1+\tan^2(\phi/2)}\right].\cos\theta + \left[\frac{2\tan(\phi/2)}{1+\tan^2(\phi/2)}\right].\sin\theta = K_1\left[\frac{1-\tan^2(\phi/2)}{1+\tan^2(\phi/2)}\right] - K_2\cos\theta + K_3$$

$$[1-\tan^2(\phi/2)].\cos\theta + [2\tan(\phi/2)]\cdot\sin\theta = [1-\tan^2(\phi/2)]K_1 - K_2\cos\theta[1+\tan^2(\phi/2)]$$
$$+ K_3[1+\tan^2(\phi/2)]$$

$$\cos\theta - \cos\theta.\tan^2(\phi/2) + 2\sin\theta.\tan(\phi/2) = K_1 - K_1\tan^2(\phi/2) - K_2\cos\theta$$
$$- K_2\cos\theta.\tan^2(\phi/2) + K_3 + K_3\tan^2(\phi/2)$$

$$-\cos\theta.\tan^2(\phi/2) + K_1\tan^2(\phi/2) + K_2\cos\theta\tan^2(\phi/2) - K_3\tan^2(\phi/2)$$
$$+ 2\sin\theta.\tan(\phi/2) = -\cos\theta + K_1 - K_2\cos\theta + K_3$$

$$-\tan^2(\phi/2)[\cos\theta - K_1 - K_2\cos\theta + K_3] + 2\sin\theta\tan(\phi/2) - K_1 - K_3$$
$$+ \cos\theta[1+K_2] = 0$$

$$[(1-K_2)\cos\theta + K_3 - K_1]\tan^2(\phi/2) + [-2\sin\theta]\tan(\phi/2)$$
$$+ [K_1 + K_3 - (1+K_2)\cos\theta] = 0$$

Taking,

$$A = (1-K_2)\cos\theta + K_3 - K_1$$

$$B = -2\sin\theta$$

$$C = K_1 + K_3 - (1+K_2)\cos\theta .$$

So,

$$A\tan^2(\phi/2) + B\tan(\phi/2) + C = 0 \quad \text{--- (7)}$$

for the above equtn we will find the roots.

$$\tan(\phi/2) = \frac{-B \pm \sqrt{B^2-4AC}}{2A}.$$

$$\boxed{\phi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right]} \quad \text{--- (8)}$$

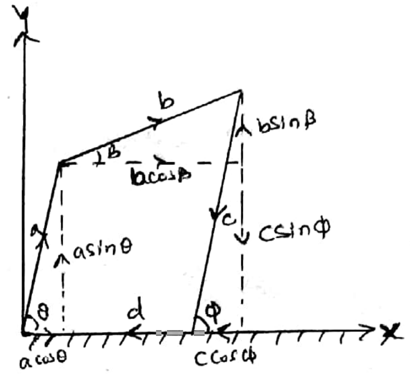
From above equtn we can find the position of output link CD (i.e angle $\phi$) if lengths and position of input line AB (i.e $\theta$) is known.

Similarly,

(ii) If the position of input link AB (i.e $\theta$) and the coupler link BC (i.e $\beta$) is required then we have to eliminate angle $\phi$.

So, the eqn can be written as

for equlbm sum ~~along~~ of component along x-axis.




$$a\cos\theta + b\cos\beta - c\cos\phi - d = 0$$

$$c\cdot\cos\phi = a\cos\theta + b\cos\beta - d$$

Squaring both sides

$$c^2\cos^2\phi = [a\cos\theta + b\cos\beta - d]^2$$

$$= a^2\cos^2\theta + b^2\cos^2\beta + d^2 + 2ab\cos\theta\cos\beta - 2ad\cos\theta - 2bd\cos\beta \quad \text{--- (1)}$$

Similarly, for equlbm sum of components along Y-axis.

$$a\sin\theta + b\sin\beta - c\sin\phi = 0$$

$$\cancel{b\sin\beta = c\sin\phi - a\sin\theta}$$

$$c\sin\phi = a\sin\theta + b\sin\beta.$$

Squaring both sides,

$$c^2\sin^2\phi = [a\sin\theta + b\sin\beta]$$

$$= a^2\sin^2\theta + b^2\sin^2\beta + 2\cdot a\cdot b\sin\theta\sin\beta \quad \text{--- (2)}$$

Adding eqn (1) and (2)

$$c^2[\cos^2\phi + \sin^2\phi] = a^2[\cos^2\theta + \sin^2\theta] + b^2[\cos^2\beta + \sin^2\beta] + d^2 + 2\cdot a\cdot b[\cos\theta\cdot\cos\beta + \sin\theta\cdot\sin\beta] - 2\cdot a\cdot d\cos\theta - 2bd\cos\beta.$$

$$c^2 = a^2 + b^2 + d^2 + 2\cdot a\cdot b[\cos\theta\cdot\cos\beta + \sin\theta\cdot\sin\beta] - 2\cdot a\cdot d\cos\theta - 2bd\cos\beta.$$

$$2ab[\cos\theta\cdot\cos\beta + \sin\theta\cdot\sin\beta] = c^2 - a^2 - b^2 - d^2 + 2ad\cos\theta + 2bd\cos\beta$$

$$\cos\theta\cdot\cos\beta + \sin\theta\cdot\sin\beta = \left[\frac{c^2 - a^2 - b^2 - d^2}{2ab}\right] + \frac{d}{b}\cos\theta + \frac{d}{a}\cos\beta.$$

let, $\frac{d}{a} = K_1$, $\frac{d}{b} = K_4$, $\frac{c^2 - a^2 - b^2 - d^2}{2ab} = K_5$

$$\cos\theta\cdot\cos\beta + \sin\theta\cdot\sin\beta = K_5 + K_4\cos\theta + K_1\cos\beta$$

$\therefore \sin\beta = \dfrac{2\tan(\beta/2)}{1+\tan^2(\beta/2)}$ and $\cos\beta = \dfrac{1-\tan^2(\beta/2)}{1+\tan^2(\beta/2)}$

Substituting in equation,

$$\cos\theta\left[\frac{1-\tan^2(\beta/2)}{1+\tan^2(\beta/2)}\right] + \sin\theta\left[\frac{2\tan(\beta/2)}{1+\tan^2(\beta/2)}\right] = K_1\left[\frac{1-\tan^2(\beta/2)}{1+\tan^2(\beta/2)}\right] + K_4\cos\theta + K_5$$

$$\cos\theta[1-\tan^2(\beta/2)] + \sin\theta[2\tan(\beta/2)] = K_1[1-\tan^2(\beta/2)] + K_4[1+\tan^2(\beta/2)]\cos\theta + K_5[1+\tan^2(\beta/2)]$$

$$\cos\theta - \cos\theta\cdot\tan^2(\beta/2) + 2\sin\theta\tan(\beta/2) = K_1 - K_1\tan^2(\beta/2) + K_4\cos\theta + K_4\cos\theta\cdot\tan^2(\beta/2) + K_5 + K_5\tan^2(\beta/2)$$

$$-\cos\theta\cdot\tan^2(\beta/2) + K_1\tan^2(\beta/2) - K_4\cos\theta\tan^2(\beta/2) - K_5\tan^2(\beta/2) + 2\sin\theta\tan(\beta/2) - K_1 - K_4\cos\theta - K_5 + \cos\theta = 0$$

$$\cancel{\tan^2(\beta/2)[\cos\theta - K_1 + K_4 + K_5] + 2\sin\theta\tan(\beta/2) - [(K_4-1)\cos\theta + K_5 + K_1] = 0}$$

$$-\tan^2(\beta/2)[\cos\theta - K_1 + K_4\cos\theta + K_5] + 2\sin\theta\tan(\beta/2) - [K_4\cos\theta - \cos\theta + K_1 + K_5] = 0$$

$$-\tan^2(\beta/2)[(K_4+1)\cos\theta + K_5 - K_1] + 2\sin\theta\tan(\beta/2) - [(K_4-1)\cos\theta + K_1 + K_5] = 0$$

$$[(K_4+1)\cdot\cos\theta + K_5 - K_1]\tan^2(\beta/2) + (-2\sin\theta)\tan(\beta/2) + [(K_4-1)\cos\theta + K_1 + K_5] = 0$$

let

$D = (K_4+1)\cos\theta + K_5 - K_1$

$E = -2\sin\theta$

$F = [(K_4-1)\cos\theta + K_5 + K_1]$

then,

$$D\tan^2(\beta/2) + E\tan(\beta/2) + F = 0$$

$$\tan(\beta/2) = \frac{-E \pm \sqrt{E^2 - 4D\cdot F}}{2D}$$

$$\boxed{\beta = 2\tan^{-1}\left(\left[\frac{-E \pm \sqrt{E^2-4DF}}{2D}\right]\right)}$$

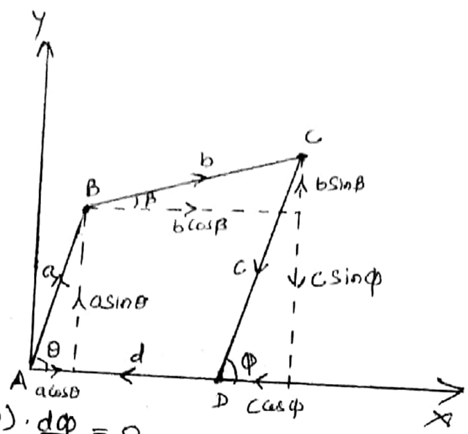
## 2. Velocity Analysis:-

For equb$^m$ of mechanism,

Sum of Component along x-axis

$a\cos\theta + b\cos\beta - c\cos\phi - d = 0$

This is displacement equation.




Differentiating Displacement equtn w.r.t time

$$-a\sin\theta \cdot \frac{d\theta}{dt} + b(-\sin\beta)\cdot\frac{d\beta}{dt} - c\cdot(-\sin\phi)\cdot\frac{d\phi}{dt} = 0$$

$$-a\omega_1 \sin\theta - \omega_2 b \sin\beta + c\cdot\omega_3 \sin\phi = 0 \quad \text{--- (1)}$$

let,
$\omega_1$ = Ang. vel. of Link AB
$\omega_1 = \frac{d\theta}{dt}$
$\omega_2$ = Ang. vel. of Link BC
$\omega_2 = \frac{d\beta}{dt}$
$\omega_3$ = Ang. vel. of Link CD
$\omega_3 = \frac{d\phi}{dt}$

Similarly displacement equtn component along Y-axis.

$a\sin\theta + b\sin\beta - c\sin\phi = 0$

to get velocity, differentiating above equtn w.r.t time.

$$a\cdot\cos\theta\cdot\frac{d\theta}{dt} + b\cdot\cos\beta\cdot\frac{d\beta}{dt} - c\cos\phi\cdot\frac{d\phi}{dt} = 0$$

$$a\cdot\omega_1\cos\theta + b\cdot\omega_2\cos\beta - c\cdot\omega_3\cos\phi = 0. \quad \text{--- (2)}$$

multiplying equtn (1) by $\cos\beta$. and equtn (2) by $\sin\beta$.

$$-a\omega_1\sin\theta\cdot\cos\beta - \cancel{b\cdot\omega_2\sin\beta\cdot\cos\beta} + c\cdot\omega_3\sin\phi\cdot\cos\beta = 0. \quad \text{--- (3)}$$

$$a\omega_1\cos\theta\cdot\sin\beta + \cancel{b\omega_2\cos\beta\cdot\sin\beta} - c\omega_3\cos\phi\cdot\sin\beta = 0. \quad \text{--- (4)}$$

Adding both equtn.

$$a\omega_1\left[\cos\theta\cdot\sin\beta - \sin\theta\cdot\cos\beta\right] + c\cdot\omega_3\left[\sin\phi\cos\beta - \cos\phi\sin\beta\right] = 0$$

$$a\omega_1\left[\sin(\beta-\theta)\right] + c\cdot\omega_3\left[\sin(\phi-\beta)\right] = 0$$

$$a\omega_1\left[\sin(\beta-\theta)\right] = -c\omega_3\left[\sin(\phi-\beta)\right]$$

$$\boxed{\omega_3 = \frac{-a\omega_1\left[\sin(\beta-\theta)\right]}{c\,\sin(\phi-\beta)}} \quad \text{--- (5)}$$

Similarly, again multiplying equtn (1) by $\cos\phi$ and equtn (2) by $\sin\phi$.

$$-a\omega_1\sin\theta\cdot\cos\phi - b\cdot\omega_2\sin\beta\cdot\cos\phi + c\omega_3\,\cancel{\sin\phi\cos\phi} = 0. \quad \text{--- (6)}$$

$$a\omega_1\cos\theta\cdot\sin\phi + b\omega_2\cos\beta\cdot\sin\phi - c\cdot\omega_3\,\cancel{\cos\phi\cdot\sin\phi} = 0. \quad \text{--- (7)}$$

Adding both equtn

$$a\omega_1\left[\sin\phi\cdot\cos\theta - \sin\theta\cdot\cos\phi\right] + b\omega_2\left[\sin\phi\cdot\cos\beta - \sin\beta\cdot\cos\phi\right] = 0$$

$$a\omega_1 \sin(\phi-\theta) + b\omega_2 \sin(\phi-\beta) = 0$$

$$b\omega_2 \sin(\phi-\beta) = -a\omega_1 \sin(\phi-\theta)$$

$$\boxed{\omega_2 = -\frac{a\omega_1 \sin(\phi-\theta)}{b\sin(\phi-\beta)}} \quad \text{—— } (8)$$

3. Acceleration Analysis: -

Considering displacement equtn (3) along x-axis i.e.

$$-a\omega_1 \sin\theta - b\omega_2 \sin\beta + c\omega_3 \sin\phi = 0.$$

let,
$\alpha_1$ = Ang. acc. of AB
$\alpha_1 = \frac{d\omega_1}{dt}$
$\alpha_2$ = Ang. acc. of BC.
$\alpha_2 = \frac{d\omega_2}{dt}$
$\alpha_3$ = Ang. acc. of CD.
$\alpha_3 = \frac{d\omega_3}{dt}$

To find acceleration, differentiating equtn w.r.t time.

$$-a\cdot\frac{d}{dt}[\omega_1 \sin\theta] - b\frac{d}{dt}[\omega_2 \sin\beta] + c\cdot\frac{d}{dt}[\omega_3 \sin\phi] = 0$$

$$-a\left[\omega_1 \cos\theta \cdot \frac{d\theta}{dt} + \sin\theta\cdot\frac{d\omega_1}{dt}\right] - b\left[\omega_2 \cos\beta\cdot\frac{d\beta}{dt} + \sin\beta\cdot\frac{d\omega_2}{dt}\right]$$
$$+ c\cdot\left[\omega_3\cdot \cos\phi\cdot\frac{d\phi}{dt} + \sin\phi\cdot\frac{d\omega_3}{dt}\right] = 0$$

$$-a\left[\omega_1^2 \cdot \cos\theta + \alpha_1\cdot \sin\theta\right] - b\left[\omega_2^2\cdot \cos\beta + \alpha_2 \sin\beta\right]$$
$$+ c\left[\omega_3^2 \cos\phi + \alpha_3 \sin\phi\right] = 0$$

$$-a\omega_1^2 \cos\theta - a\alpha_1 \sin\theta - b\omega_2^2 \cos\beta - b\alpha_2 \sin\beta + c\cdot\omega_3^2 \cos\phi$$
$$+ c\cdot\alpha_3 \sin\phi = 0 \quad \text{——— } (T)$$

Similarly takin equtn (2) of displacement analaysis along Y-axis i.e.

$$a\omega_1 \cos\theta + b\omega_2 \cos\beta - c\omega_3 \cos\phi = 0$$

Differentiating above equtn w.r.t time to find acceleration

$$a\cdot\frac{d}{dt}[\omega_1 \cos\theta] + b\cdot\frac{d}{dt}[\omega_2 \cdot \cos\beta] - c\cdot\frac{d}{dt}[\omega_3 \cos\phi] = 0$$

$$a\cdot\left[\omega_1 \cdot \frac{d}{dt}\cos\theta + \cos\theta\cdot\frac{d}{dt}\omega_1\right] + b\left[\omega_2\frac{d}{dt}\cos\beta + \cos\beta\cdot\frac{d}{dt}\omega_2\right]$$
$$- c\cdot\left[\omega_3\cdot\frac{d}{dt}\cos\phi + \cos\phi\cdot\frac{d}{dt}\omega_3\right] = 0$$

$$a\left[\omega_1(-\sin\theta)\cdot\frac{d\theta}{dt} + \cos\theta\cdot\alpha_1\right] + b\left[\omega_2(-\sin\beta)\cdot\frac{d\beta}{dt} + \cos\beta\cdot\alpha_2\right]$$
$$- c\cdot\left[\omega_3(-\sin\phi)\cdot\frac{d\phi}{dt} + \cos\phi\cdot\alpha_3\right] = 0$$

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$$a\left[-\omega_1^2 \sin\theta + \alpha_1 \cos\theta\right] + b\left[-\omega_2^2 \sin\beta + \alpha_2 \cos\beta\right]$$
$$- c\cdot\left[-\omega_3^2 \sin\phi + \alpha_3 \cos\phi\right] = 0$$

$$-a\omega_1^2 \sin\theta + a\alpha_1 \cos\theta - b\omega_2^2 \sin\beta + b\alpha_2 \cos\beta + c\cdot\omega_3^2 \sin\phi - c\alpha_3 \cos\phi = 0 \quad \text{—— (2)}$$

multiplying equtn (1) of acceleration analysis by $\cos\phi$ and equtn (2) by $\sin\phi$.

$$-a\omega_1^2 \cos\theta\cdot\cos\phi - a\alpha_1 \sin\theta\cdot\cos\phi - b\omega_2^2 \cos\beta\cdot\cos\phi - b\alpha_2 \sin\beta\cdot\cos\phi$$
$$+ c\cdot\omega_3^2 \cos\phi\cdot\cos\phi + c\alpha_3 \sin\phi\cdot\cos\phi = 0 \quad \text{—— (3)}$$

$$-a\omega_1^2 \sin\theta\cdot\sin\phi + a\alpha_1 \cos\theta\cdot\sin\phi - b\omega_2^2 \sin\beta\cdot\sin\phi + b\alpha_2 \cos\beta\cdot\sin\phi \quad \text{—— (4)}$$
$$+ c\omega_3^2 \sin\phi\cdot\sin\phi - c\alpha_3 \cos\phi\cdot\sin\phi = 0$$

Adding equtn (3) and (4)

$$-a\omega_1^2 \cos\theta\cdot\cos\phi - a\alpha_1 \sin\theta\cdot\cos\phi - b\omega_2^2 \cos\beta\cdot\cos\phi - b\alpha_2 \sin\beta\cdot\cos\phi$$
$$+ c\cdot\omega_3^2 \cos^2\phi + c\cdot\alpha_3 \sin\phi\cdot\cos\phi - a\omega_1^2 \sin\theta\cdot\sin\phi + a\alpha_1 \cos\theta\cdot\sin\phi$$
$$- b\omega_2^2 \sin\beta\cdot\sin\phi + b\alpha_2 \cos\beta\cdot\sin\phi + c\omega_3^2 \sin^2\phi - c\alpha_3 \cos\phi\cdot\sin\phi = 0$$

$$-a\omega_1^2\left[\cos\theta\cdot\cos\phi + \sin\theta\cdot\sin\phi\right] - b\omega_2^2\left[\cos\beta\cos\phi + \sin\beta\cdot\sin\phi\right] + c\omega_3\left[\cos^2\phi + \sin^2\phi\right]$$
$$+ a\alpha_1\left[\cos\theta\cdot\sin\phi - \sin\theta\cdot\cos\phi\right] + b\cdot\alpha_2\left[\cos\beta\cdot\sin\phi - \sin\beta\cdot\cos\phi\right]$$

$$-a\omega_1^2 \cos(\phi-\theta) + a\alpha_1 \sin(\phi-\theta) - b\omega_2^2 \cos(\phi-\beta) + b\alpha_2 \sin(\phi-\beta) + c\omega_3^2 = 0$$

$$b\alpha_2 \sin(\phi-\beta) = -a\alpha_1 \sin(\phi-\theta) + a\omega_1^2 \cos(\phi-\theta) + b\omega_2^2 \cos(\phi-\beta) - c\omega_3^2$$

$$\boxed{\alpha_2 = \frac{-a\alpha_1 \sin(\phi-\theta) + a\omega_1^2 \cos(\phi-\theta) + b\omega_2^2 \cos(\phi-\beta) - c\omega_3^2}{b \sin(\phi-\beta)}} \quad \text{—— (5)}$$

Similarly, multiplying equtn (1) of acceleration analysis by $\cos\beta$ and equtn (2) by $\sin\beta$.

$$-a\omega_1^2 \cos\theta\cdot\cos\beta - a\alpha_1 \sin\theta\cdot\cos\beta - b\omega_2^2 \cos\beta\cdot\cos\beta - b\alpha_2 \sin\beta\cdot\cos\beta$$
$$+ c\cdot\omega_3^2 \cos\phi\cdot\cos\beta + c\cdot\alpha_3 \sin\phi\cdot\cos\beta = 0 \quad \text{—— (6)}$$

$$-a\omega_1^2 \sin\theta\cdot\sin\beta + a\alpha_1 \cos\theta\cdot\sin\beta - b\omega_2^2 \sin\beta\cdot\sin\beta + b\alpha_2 \cos\beta\cdot\sin\beta$$
$$+ c\omega_3^2 \sin\phi\cdot\sin\beta - c\alpha_3 \cos\phi\cdot\sin\beta = 0 \quad \text{—— (7)}$$

Adding equtn (6) and (7).

$$-a\omega_1^2 \cos\theta\cdot\cos\beta - a\alpha_1 \sin\theta\cdot\cos\beta - b\omega_2^2 \cos^2\beta - b\alpha_2 \sin\beta\cos\beta$$
$$+ c\omega_3^2 \cos\phi\cdot\cos\beta + c\alpha_3 \sin\phi\cos\beta - a\omega_1^2 \sin\theta\cdot\sin\beta + a\alpha_1 \cos\theta\cdot\sin\beta$$
$$- b\omega_2^2 \sin^2\beta + b\alpha_2 \cos\beta\cdot\sin\beta + c\omega_3^2 \sin\phi\sin\beta - c\alpha_3 \cos\phi\cdot\sin\beta = 0$$

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$$-a\omega_1^2[\cos\beta\cdot\cos\theta + \sin\beta\sin\theta] + a\alpha_1[\sin\beta\cdot\cos\theta - \cos\beta\cdot\sin\theta]$$

$$-b\omega_2^2[\cos^2\beta + \sin^2\beta] + c\omega_3^2[\cos\phi\cos\beta + \sin\phi\sin\beta]$$

$$+c\alpha_3[\sin\phi\cos\beta - \cos\phi\sin\beta] = 0$$

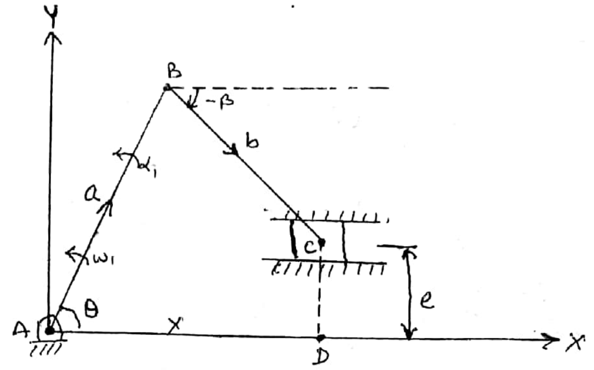
$$-a\omega_1^2\cos(\beta-\theta) + a\alpha_1\sin(\beta-\theta) - b\omega_2^2 + c\omega_3^2[\cos(\phi-\beta)]$$

$$+ c\alpha_3\sin(\phi-\beta) = 0$$

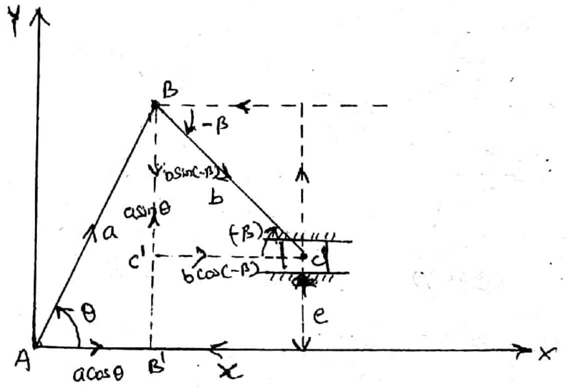
$$c\alpha_3\sin(\phi-\beta) = a\omega_1^2\cos(\beta-\theta) - a\alpha_1\sin(\beta-\theta) + b\omega_2^2 - c\omega_3^2\cos(\phi-\beta)$$

$$\boxed{\alpha_3 = \frac{a\omega_1^2\cos(\beta-\theta) - a\alpha_1\sin(\beta-\theta) + b\omega_2^2 - c\omega_3^2\cos(\phi-\beta)}{c\cdot\sin(\phi-\beta)}}$$

# *Computer Aided Analysis of slider crank mechanism :-




let,
$\omega_1$ = Ang. velocity of crank (S)
$\alpha_1$ = — " acc<sup></sup>n — " — " — (S)




## 1. Displacement Analysis :-

For equilibrium, sum of components along X-axis and Y-axis must be zero, so ~~Co~~ sum of component along x-axis. i.e.

$$a\cos\theta + b\cos(-\beta) - x = 0$$

$$b\cos\beta = x - a\cos\theta$$

| |
|---|
| $\because \cos(-\theta) = \cos\theta$ |
| $\sin(-\theta) = -\sin\theta$ |

Squaring both sides

$$b^2\cos^2\beta = (x - a\cos\theta)$$

$$b^2\cos^2\beta = x^2 + a^2\cos^2\theta - 2xa\cos\theta \qquad \text{— (1)}$$

Similarly taking sum of components along Y-axis.

$$a\sin\theta - b\sin(-\beta) \mp e = 0$$

$$a\sin\theta + b\sin\beta \mp e = 0$$

$$b\sin\beta = e - a\sin\theta$$

Squaring both sides.

$$b^2\sin^2\beta = (e - a\sin\theta)^2$$

$$b^2\sin^2\beta = e^2 + a^2\sin^2\theta - 2ae\sin\theta \qquad \text{— (2)}$$

Adding eqn (1) and (2).

$$b^2[\cos^2\beta + \sin^2\beta] = x^2 + a^2\cos^2\theta - 2xa\cos\theta + e^2 + a^2\sin^2\theta - 2ae\sin\theta$$

$$b^2 = x^2 + a^2 + e^2 - 2xa\cos\theta - 2ae\sin\theta .$$

$$x^2 + (-2a\cos\theta)x + a^2 + e^2 - b^2 - 2ea\sin\theta = 0$$

$$\boxed{x^2 + K_1 x + K_2 = 0}$$

where,

$$K_1 = -2a\cos\theta$$

$$K_2 = a^2 - b^2 + e^2 - 2ea\sin\theta$$

$$\boxed{x = \frac{-K_1 \pm \sqrt{K_1^2 - 4(K_2)}}{2}}$$

Similarly if we have to find angular displacement $\beta$ than, Considering sum along Y-axis,

$$a\sin\theta - b\sin(-\beta) - e = 0$$

$$a\sin\theta + b\sin\beta - e = 0$$

$$b\sin\beta = e - a\sin\theta$$

$$\sin\beta = \frac{e - a\sin\theta}{b}$$

$$\boxed{\beta = \sin^{-1}\left[\frac{e - a\sin\theta}{b}\right]}$$

when $e = 0$, i.e slider is along the axis of rotation.

$$\boxed{\beta = \sin^{-1}\left[\frac{-a\sin\theta}{b}\right]}$$

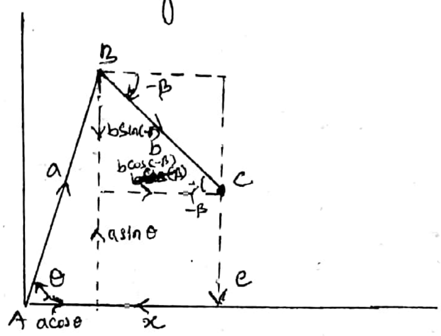
2. Velocity Analysis:-

let, $\omega_1$ = Angular vel. of Crank AB = $\frac{d\theta}{dt}$

$\omega_2$ = — " — " — " Connecting rod BC = $\frac{d\beta}{dt}$

$V_s$ = Linear — " — " — slider = $\frac{dx}{dt}$

So taking the sum of components along x-axis.




$a\cos\theta + b\cos(-\beta) - x = 0$

$b\cos\beta = x - a\cos\theta$

Differentiating above equtn w.r.t time

$b.\frac{d}{dt}\cos\beta = \frac{dx}{dt} - a\frac{d}{dt}\cos\theta$

$b.(-\sin\beta)\frac{d\beta}{dt} = V_s - a(-\sin\theta).\frac{d\theta}{dt}$

$-b\sin\beta(\omega_2) = V_s + a\omega_1\sin\theta.$

$-a\omega_1\sin\theta - b\omega_2\sin\beta - \frac{dx}{dt} = 0$ ——— (1)

Similarly taking the sum of Components along Y-axis.

$a\sin\theta - b\sin(-\beta) - e = 0$

$a\sin\theta + b\sin\beta - e = 0$

$b\sin\beta = e - a\sin\theta$

Differentiating above equtn w.r.t time

$b.\cos\beta.\frac{d\beta}{dt} = 0 - a\cos\theta.\frac{d\theta}{dt}$

$b.\omega_2\cos\beta = -a\omega_1\cos\theta$

$a\omega_1\cos\theta + b\omega_2\cos\beta = 0$ ——— (2)

Multiplying equtn (1) by $\cos\beta$ and equtn (2) by $\sin\beta$.

$-a\omega_1\sin\theta.\cos\beta - b\omega_2\sin\beta.\cos\beta - \cos\beta.\frac{dx}{dt} = 0$ — (3)

$a\omega_1\cos\theta.\sin\beta + b\omega_2\cos\beta.\sin\beta = 0$ ——— (4)

Adding equtn (3) and (4).

$a\omega_1[\cos\theta.\sin\beta - \sin\theta\cos\beta] - \frac{dx}{dt}.\cos\beta = 0.$

$$a\omega_1 \sin(\beta-\theta) = \frac{dx}{dt} \cdot \cos\beta$$

$$\boxed{\frac{dx}{dt} = \frac{a\omega_1 \sin(\beta-\theta)}{\cos\beta}}$$ Linear equtn of slider for velocity ($V_s$)

Similarly after differentiating equtn of sum of component in Y w.r.t i.e equtn equtn (2)

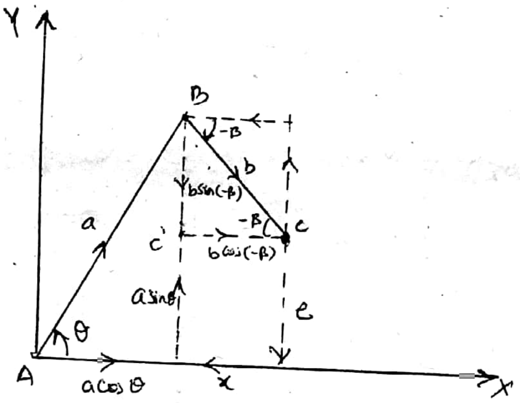
$$a\omega_1 \cos\theta + b\omega_2 \cos\beta = 0$$

$$a\omega_1 \cos\theta = -b\omega_2 \cos\beta$$

$$\boxed{\omega_2 = \frac{-a\omega_1 \cos\theta}{b\cos\beta}}$$

This is the equtn for the angular velocity of connecting rod.

3. <u>Acceleration Analysis</u> :-




$\alpha_1$ = Angular acceleration of the Crank $= \frac{d\omega_1}{dt}$

$\alpha_2$ = Angular acceleration of the Connecting rod $= \frac{d\omega_2}{dt}$

$\alpha_s$ = Linear acceleration of the slider $= \frac{d^2x}{dt^2}$

Considering velocity equtn of all Components along X-axis :-

$$-a\omega_1 \sin\theta - b\omega_2 \sin\beta - \frac{dx}{dt} = 0$$

Differentiating velocity equtn w.r.t time to get acceleration equtn.

$$-a\left[\omega_1 \cos\theta \cdot \frac{d\theta}{dt} + \sin\theta \cdot \frac{d\omega_1}{dt}\right] - b\left[\omega_2 \cos\beta \cdot \frac{d\beta}{dt} + \sin\beta \cdot \frac{d\omega_2}{dt}\right] - \frac{d^2x}{dt^2} = 0.$$

$$-a\left[\alpha_1 \sin\theta + \omega_1^2 \cos\theta\right] - b\left[\alpha_2 \sin\beta + \omega_2^2 \cos\beta\right] - \frac{d^2x}{dt^2} = 0 \quad \text{--- (1)}$$

Similarly ~~differentiating~~ considering velocity in y-axis i.e.

$$a\omega_1 \cos\theta + b\omega_2 \cos\beta = 0$$

Differentiating above equn w.r.t time to get acceleration.

$$a\left[\omega_1 \cdot (-\sin\theta) \cdot \frac{d\theta}{dt} + \cos\theta \cdot \frac{d\omega_1}{dt}\right] + b\left[\omega_2 (-\sin\beta) \cdot \frac{d\beta}{dt} + \sin\beta \cdot \frac{d\omega_2}{dt}\right] = 0$$

$$a\left[\alpha_1 \cdot \cos\theta - \omega_1^2 \sin\theta\right] + b\left[\alpha_2 \cos\beta - \omega_2^2 \sin\beta\right] = 0 \quad \text{--- (2)}$$

multiplying equn (1) by $\cos\beta$ and equn (2) by $\sin\beta$.

$$-a\left[\alpha_1 \sin\theta \cdot \cos\beta + \omega_1^2 \cos\theta \cdot \cos\beta\right] - b\left[\alpha_2 \sin\beta \cdot \cos\beta + \omega_2^2 \cos^2\beta\right] - \cos\beta \cdot \frac{d^2x}{dt^2} = 0 \quad \text{--- (3)}$$

$$a\left[\alpha_1 \cos\theta \cdot \sin\beta - \omega_1^2 \sin\theta \cdot \sin\beta\right] + b\left[\alpha_2 \cos\beta \cdot \sin\beta - \omega_2^2 \sin^2\beta\right] = 0 \quad \text{--- (4)}$$

Adding equn (3) and (4).

$$a\left[\alpha_1 (\cos\theta \cdot \sin\beta - \sin\theta \cdot \cos\beta) - \omega_1^2 (\cos\beta \cdot \cos\theta + \sin\beta \cdot \sin\theta)\right]$$
$$- b\omega_2 (\cos^2\beta + \sin^2\beta) - \frac{d^2x}{dt^2} \cdot \cos\beta = 0$$

$$a\alpha_1 \sin(\beta-\theta) - a\omega_1^2 \cos(\beta-\theta) - b\omega_2 - \frac{d^2x}{dt^2} \cdot \cos\beta = 0$$

$$\boxed{\frac{d^2x}{dt^2} = \frac{a\alpha_1 \sin(\beta-\theta) - a\omega_1^2 \cos(\beta-\theta) - b\omega_2^2}{\cos\beta}}$$

This equtn gives the linear ~~equ~~ acceleration of slider ($a_s$)

Similarly from equtn (2),

$$b\alpha_2 \cos\beta - b\omega_2^2 \sin\beta = a\left[\omega_1^2 \sin\theta - \alpha_1 \cos\theta\right]$$

$$\boxed{\alpha_2 = \frac{a\left[\alpha_1 \cos\theta - \omega_1^2 \sin\theta\right] - b\omega_2^2 \sin\beta}{b\cos\beta}}$$

Angular acceleration of connecting rod.